## BCA Semester-IV (Hons.) Examination, 2022-23 BACHELOR OF COMPUTER APPLICATION

Course ID : 43314
Course Code : GE-04

## Course Title : Mathematics-III

Time : 3 Hours
Full Marks: 80
The figures in the right-hand margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

## GROUP-A

1. Choose the best alternative from the following options for each questions:
$1 \times 10=10$
a) For any two events A and B it is given that $\mathrm{P}\left(\mathrm{A}^{\mathrm{C}}\right)=0.4, \mathrm{P}(\mathrm{AB})=0.3$ and $\mathrm{P}(\mathrm{A} / \mathrm{B})=0.75$. The value of $\mathrm{P}\left(\mathrm{B}^{\mathrm{C}}\right)$ is
i) 0.9
ii) 0.6
iii) 1.5
iv) 0.54
b) ${ }^{\mathrm{n}+1} \mathrm{C}_{\mathrm{r}}=$
i) ${ }^{n} \mathrm{C}_{\mathrm{r}}+{ }^{\mathrm{n}} \mathrm{C}_{\mathrm{r}-1}$
iii) ${ }^{n-1} \mathrm{C}_{\mathrm{r}}+{ }^{\mathrm{n}} \mathrm{C}_{\mathrm{r}-1}$
ii) ${ }^{n} C_{r+1}+{ }^{n} C_{r-1}$
iv) ${ }^{n} \mathrm{C}_{\mathrm{r}}+{ }^{\mathrm{n}} \mathrm{C}_{\mathrm{r}-2}$
c) Mean Deviation is minimum when taken about
i) Mean
ii) Harmonic Mean
iii) Median
iv) Mode
d) In sampling theory, SRSWR means
i) Simple Random Sampling with Reasoning
ii) Startified Random Sampling with Replacement
iii) Simple Replacement Sampling with Reasoning
iv) Simple Random Sampling with Replacement.
e) Significance level of a test is associated with
i) Power of the test
ii) Type II error
iii) Type I error
iv) None of these
f) An unbiased die is thrown. Then the mathematical expectation of the number on the face appeared is
i) $1 / 2$
ii) $7 / 2$
iii) $13 / 2$
iv) $3 / 2$
g) Which one of the following is correct?
i) Bisection method is an iterative method
ii) Regula Falsi method is direct method
iii) Secant method is direct method
iv) Newton Raphson method is not iterative method
h) Gauss Seidel iterative method can be used for solving a set of
i) Linear differential equation only
ii) Linear algebraic equations only
iii) Both linear and nonlinear algebraic equations
iv) Both linear and nonlinear differential equations
i) In Simpson's $1 / 3$ rule, the actual area under the curve is replaced by
i) A parabola
ii) a straight line
iii) any curve
iv) a trapezium
j) Lagrange's interpolation formula can be used for
i) Equi-spaced argument values only
ii) Equi-spaced and non-equispaced argument values
iii) Non-equispaced argument values only
iv) None of the above.

## GROUP-B

2. Answer any ten of the following questions:

$$
2 \times 10=20
$$

a) Prove that $\mid \mathrm{E}\{\mathrm{g}(\mathrm{X}\} \mid \leq \mathrm{E}\{|\mathrm{g}(\mathrm{X})|\}$ where $\mathrm{g}(\mathrm{X})$ is any function of continuous random variable X .
b) The coefficient of variation is 40 and the arithmetic mean is 30 ; find the standard deviation.
c) What do you mean by Scatter diagram?
d) What is the difference between linear regression and multiple regression?
e) A simple random sample of size 5 is drawn without replacement from a finite population consisting of 41 units. If the population standard deviation is 6.25 , what is the standard error of sample mean? (Use finite population correction).
f) Write down any two properties of Student's $t$ distribution.
g) Prove that $\Delta(\mathrm{x}+\cos \mathrm{x})=h-2 \sin \left(x+\frac{h}{2}\right) \sin \frac{h}{2}$. ( $h$ is the step length).
h) Why does one need to use numerical method instead of analytical method for integration?
i) Examine whether the Gauss Seidel iteration method is applicable or not for the following system of equation:

$$
\begin{aligned}
& x+5 y+z=1 \\
& 4 x+3 y+2 z=-3 \\
& x+2 y-3 z=2
\end{aligned}
$$

j) What are the drawbacks of Taylor's series method for first order ODE?
k) Write down the condition of convergence of $\mathrm{N}-\mathrm{R}$ method. Mention that whether the condition is sufficient or necessary or both.

1) If the letters of the word 'MOTHER' are arranged at random, find the probability that the vowels will be next to each other.
m) What is the difference between Type I and Type II errors in the Hypothesis theory?
n) What do you mean by Statistical Inference?
o) 100 liters of water are supposed to be polluted with $10^{5}$ bacteria. Find the probability that a sample of 1 c.c. of the same water is free from bacteria.

## GROUP-C

3. Answer any four of the following questions:

$$
5 \times 4=20
$$

a) Define probability distribution function and prove that the distribution function is leftly discontinuous.
b) Find the arithmetic mean and standard deviation of the first $n$ natural numbers.
c) Write down the p.d.f. of normal distribution and find $P(\backslash X-50 \mid \leq 20)$ where $X$ is a random variable following normal $\mathrm{N}(50,20)$ distribution.
$\left[\right.$ Given $\left.\frac{1}{\sqrt{2 \pi}} \int_{-\infty}^{1} e^{-\frac{x^{2}}{2}} d x=0.8413\right]$.
d) i) Define correlation and covariance.
ii) For random variables X and Y with the same mean, the two regression equations are $Y=a X+b$ and $X=\alpha Y+\beta$. Show that $\frac{b}{\beta}=\frac{1-a}{1-\alpha}$.
e) Derive the Newton's forward interpolation formula.
f) Explain Euler's method for solving first order ODE with its advantages and drawbacks.

## GROUP-D

4. Answer any three of the following questions:

$$
10 \times 3=30
$$

a) i) Given $r_{12}=-0.7, r_{13}=0.6$, and $r_{23}=0.52$, compute the correlation between $\mathrm{X}_{1}$ and $\mathrm{X}_{2}$ while controlling for $\mathrm{X}_{3} . \mathrm{N}=40$ for all r .
ii) For a symmetrical distribution A.M = Median $=8.5$. The semi-interquartile range is 4 . Find the first and third quartile.
iii) If $\theta$ be the acute angle between the two regression lines, then show that

$$
\tan \theta=\frac{1-r^{2}}{r} \frac{S_{x} S_{y}}{S_{x}^{2}+S_{y}^{2}}
$$

where $r$ is the correlation coefficient and $S_{x}$ and $S_{y}$ are the respective S.D. $3+3+4$
b) i) Prove that $f(\mathrm{x})=\frac{1}{2} \mathrm{e}^{-|\mathrm{x}|},-\infty<\mathrm{x}<\infty$ is a possible probability density function. Find the corresponding distribution function.
ii) Nine patients to whom a certain drug was administrated, registered the following rise of blood pressure: $3,7,4,-1,-3,6,-4,1$, 5. Test the hypothesis that the drug does not rise blood pressure at $10 \%$ level of
significance. (Assume that the sample from a normal population)
$[\mathrm{P}(\mathrm{t}>1.86)=0.05$ for 8 degrees of freedom.]
c) i) Deduce the condition of convergency of Fixed-point iteration method.
ii) Evaluate $\int_{-1}^{3}|x| d x$ numerically by Trapezoidal rule taking four equal subintervals.
iii) The following values of the function $f(\mathrm{x})$ for value of x are given: $f(1)=4, f(2)=5$, $f(7)=5, f(8)=4$. Find the values of $f(6)$ and also the value of x for which $f(\mathrm{x})$ is maximum or minimum. $3+3+4$
d) i) Describe Regula Falsi method.
ii) Find the error term in Simpson's $1 / 3$ integration formula.
e) i) Find the mean and variance of normal $\mathrm{N}(\mathrm{m}, \sigma)$ distribution.
ii) For a normal $\mathrm{N}(\mathrm{m}, \sigma)$ population, find the sampling distribution of $\mathrm{s}^{2}$, where $(n-1) s^{2}=n S^{2}, S^{2}$ being the sample variance.

